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AN UPPER BOUND ON P-WAVE CHARMONIUM PRODUCTION VIA THE COLOR-OCTET MECHANISM*

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ABSTRACT

A factorization theorem for P-wave quarkonium production, recently derived by Bodwin, Braaten, Yuan and Lepage, is applied to $\Upsilon \to \chi_{cJ} + X$, where χ_{cJ} labels the ${}^3P_J$ charmonium states. The widths for χ_{cJ} production through color-singlet P-wave and color-octet S-wave $c\bar{c}$ subprocesses are computed each to leading order in α_s. Experimental data on $\Upsilon \to J/\psi + X$ is used to obtain an upper bound on a nonperturbative parameter (related to the probability for color-octet S-wave $c\bar{c}$ hadronization into P-wave charmonium) that enters into the factorization theorem.

Factorization theorems play a basic role in perturbative QCD calculations of many hadronic processes. A well known factorization theorem for the decay and production of S-wave quarkonium follows from a nonrelativistic description of heavy quark-antiquark ($Q\overline{Q}$) binding [1]. Nonperturbative effects are factored into $R_S(0)$, the nonrelativistic wave function at the origin, leaving a hard $Q\overline{Q}$ subprocess matrix element that can be calculated in perturbation theory.

Remarkably, the correct factorization theorems for the decay [2] and production [3] of P-wave quarkonium have only recently been derived. These new theorems resolve a long standing problem regarding infrared divergences which appear in some cases to leading order in the rates for P-wave $Q\overline{Q}$ states [4]. In previous phenomenological calculations, the divergence was replaced by a logarithm of a soft binding scale [4, 1]. However, a rigorous calculation requires that one consider additional components of the Fock space for P-wave quarkonium, such as $|Q\overline{Q}g\rangle$, where the $Q\overline{Q}$ pair is in a color-octet S-wave state, and g is a soft gluon [2, 3].

A renewed study of the decay and production of P-wave quarkonium is therefore of considerable interest, since one may gain new information on a nonperturbative sector of QCD that has largely been neglected in the quark model description of heavy quarkonium. This is also of practical consequence; for example, J/ψ production provides a clean experimental signature for many important processes, and P-wave charmonium states have appreciable branching fractions to J/ψ.

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In this paper the factorization theorem for P-wave quarkonium production is applied to $\Upsilon \to \chi_{cJ}+X$, where χ_{cJ} labels the ^3P_J charmonium states. The widths for χ_{cJ} production through color-singlet P-wave and color-octet S-wave $c\bar{c}$ subprocesses are computed each to leading order in α_s. Experimental data on $\Upsilon \to J/\psi+X$ is used to obtain an upper bound on a nonperturbative parameter (related to the probability for color-octet S-wave $c\bar{c}$ hadronization into P-wave charmonium) that enters into the factorization theorem. The bound obtained here adds to the limited information so far available on the color-octet mechanism for P-wave quarkonium production. The color-octet component in P-wave decay was estimated in Ref. [2] from measured decay rates of the χ_{c1} and χ_{c2}. A rough estimate of the color-octet component in P-wave charmonium production was obtained in Ref. [3] from data on B meson decays; however, an accurate determination in that case requires a calculation of next-to-leading order QCD corrections to the color-singlet component of $B \to \chi_{cJ}+X$, which is so far unavailable [3].

The factorization theorem for P-wave quarkonium production has two terms, and in the case of Υ decay takes the form:

$$\begin{aligned}\Gamma(\Upsilon \to \chi_{cJ}+X) \;=\;& H_1\hat{\Gamma}_1(\Upsilon \to c\bar{c}(^3P_J)+X;\mu) \\ &+ (2J+1)H_8'(\mu)\hat{\Gamma}_8(\Upsilon \to c\bar{c}(^3S_1)+X). \end{aligned} \tag{1}$$

$\hat{\Gamma}_1$ and $\hat{\Gamma}_8$ are hard subprocess rates for the production of a $c\bar{c}$ pair in color-singlet P-wave and color-octet S-wave states respectively. The quarks are taken to have vanishing relative momentum. The nonperturbative parameters H_1 and H_8' are proportional to the probabilities for these $c\bar{c}$ configurations to hadronize into a color-singlet P-wave bound state. H_1, H_8' and $\hat{\Gamma}_8$ are independent of J. This factorization theorem is valid to all orders in α_s and to leading order in v^2, where v is the typical center-of-mass velocity of the heavy quarks. The hard subprocess rates are free of infrared divergences. $\hat{\Gamma}_1$ and H_8' depend on an arbitrary factorization scale μ in such a way that the physical decay rate is independent of μ. In order to avoid large logarithms of m_Υ/μ in $\hat{\Gamma}_1$, μ of $O(m_\Upsilon)$ should be used.

H_1 can be expressed in terms of the P-wave color-singlet $c\bar{c}$ wave function, $H_1 \approx 9|R_P'(0)|^2/(2\pi m_c^4) \approx 15$ MeV, where the numerical estimate was obtained in Ref. [2] from measured decay rates of the χ_{c1} and χ_{c2}. H_8' cannot be rigorously expressed perturbatively in terms of R_P, since it accounts for radiation of a soft gluon by a color-octet $c\bar{c}$ pair. The scale dependence of $H_8'(\mu)$ is determined by a renormalization-group equation [2, 5] which (at leading order in $\alpha_s(\mu)$) gives [3]:

$$H_8'(m_b) = H_8'(\mu_0) + \left[\frac{16}{27\beta_3}\ln\left(\frac{\alpha_s(\mu_0)}{\alpha_s(m_c)}\right) + \frac{16}{27\beta_4}\ln\left(\frac{\alpha_s(m_c)}{\alpha_s(m_b)}\right)\right] H_1 \tag{2}$$

(for $\mu_0 < m_c$), where $\beta_n = (33-2n)/6$. If $H_8'(\mu_0)$ is neglected in the limit of large m_b one obtains $H_8'(m_b) \approx 3$ MeV, using $\alpha_s(\mu_0) \sim 1$ [3]. While one might not expect the physical value of m_b to be large enough to neglect $H_8'(\mu_0)$, an estimate for $H_8'(m_b)$ obtained in Ref. [3] from experimental data on B meson decays is consistent with the above result.

A calculation of $\hat{\Gamma}_1$ and $\hat{\Gamma}_8$ in Eq. (1) each to leading order in α_s can be obtained from a calculation of the infrared divergent width $\Gamma_{\rm div}$ for $\Upsilon \to c\bar{c}(^3P_J)+ggg$, where the $c\bar{c}$ pair is in a color-singlet P-wave state:

$$\begin{aligned}&\Gamma_{\rm div}(\Upsilon \to c\bar{c}(^3P_J) + ggg; \mu_0) \equiv \\ &\qquad \frac{20\alpha_s^5}{3^7\pi^3}\frac{G_1^\Upsilon}{m_\chi}\left[\mathcal{F}_{1J}(\mu) + (2J+1)\frac{16}{27\pi}\ln\left(\frac{\mu}{\mu_0}\right)\mathcal{F}_8\right] H_1. \end{aligned} \tag{3}$$

$\mathcal{F}_{1J}$ and $\mathcal{F}_8$ are dimensionless infrared-finite form factors. μ_0 is an infrared cutoff on the energy of soft gluons, and μ is an arbitrary factorization scale [the μ dependence of $\mathcal{F}_{1J}$ exactly cancels that of the explicit logarithm in Eq. (3)]. The constants in Eq. (3) include a color-factor of 5/216 and phase space factors, including 1/3 for Υ spin-averaging, and 1/3! for the phase space of the three indistinguishable gluons [cf. Eq. (9) below]. G_1^Υ is related to the usual S-wave $b\bar{b}$ nonrelativistic wave function, $G_1^\Upsilon \approx 3|R_S^\Upsilon(0)|^2/(2\pi m_b^2) \approx 108$ MeV, where the numerical value is obtained from the electronic decay rate of the Υ [6].

The hard subprocess rates of Eq. (1) are identified from $\Gamma_{\rm div}$ by using the perturbative expression for the infrared divergence in H_8', obtained by neglecting the running of the coupling [3]: $H_8'(\mu) \sim (16/27\pi)\alpha_s \ln(\mu/\mu_0) H_1$. Thus:

$$\hat{\Gamma}_1(\Upsilon \to c\bar{c}(^3P_J) + ggg; \mu) = \frac{20\alpha_s^5}{3^7\pi^3}\frac{G_1^\Upsilon}{m_\chi}\mathcal{F}_{1J}(\mu), \tag{4}$$

and

$$\hat{\Gamma}_8(\Upsilon \to c\bar{c}(^3S_1) + gg) = \frac{20\alpha_s^4}{3^7\pi^3}\frac{G_1^\Upsilon}{m_\chi}\mathcal{F}_8. \tag{5}$$

Note that $\hat{\Gamma}_1$ is suppressed by $O(\alpha_s)$ compared to $\hat{\Gamma}_8$. However, the nonperturbative parameters H_1 and H_8' which accompany these subprocess rates in Eq. (1) are independent, hence $\alpha_s H_1$ need not be small compared to H_8' [2, 3]. We therefore proceed to calculate $\hat{\Gamma}_1$ and $\hat{\Gamma}_8$ each to leading order; all further corrections to P-wave production are then guaranteed to be suppressed by at least one power of α_s compared to what is included here.

In order to extract $\mathcal{F}_{1J}$ and $\mathcal{F}_8$ individually, it is necessary to explicitly identify the infrared logarithm in the calculation of $\Gamma_{\rm div}$. This can be done analytically, as described in the following. There are 36 $O(\alpha_s^5)$ diagrams contributing to $\Gamma_{\rm div}$. One of these is shown in Fig. 1. Define the invariant amplitude $\mathcal{M}_J(2,3;1)$ corresponding to the sum of all Feynman diagrams where gluon "1" is radiated from the charm quark line. The amplitude is readily computed using expressions for S- and P-wave $Q\overline{Q}$ currents given in Ref. [7][†]

$$\mathcal{M}_J(2,3;1) \equiv -\frac{m_\Upsilon m_\chi B_\mu(2,3) C_J^\mu(1)}{[(k_2+k_4)\cdot k_3][(k_3+k_4)\cdot k_2][(k_2+k_3)\cdot k_4]\,k_4^2\,(k\cdot k_1)^2}, \tag{6}$$

[†]Overall factors in the quark currents including couplings, color amplitudes, and wave functions have been accounted for in Eq. (3).

Figure 1: One of the 36 $O(\alpha_s^5)$ diagrams contributing to $\Upsilon \to c\bar{c}(^3P_J) + ggg$.

where

$$\begin{aligned}
\epsilon_4^\mu B_\mu(2,3) \quad = \quad & \{\epsilon_4 \cdot \epsilon_2[-k_4 \cdot k_3 \epsilon_3 \cdot k_2 \epsilon_0 \cdot k_4 - k_2 \cdot k_3 \epsilon_3 \cdot k_4 \epsilon_0 \cdot k_2 - k_4 \cdot k_3 k_2 \cdot k_3 \epsilon_0 \cdot \epsilon_3] \\
& + \epsilon_0 \cdot \epsilon_3[k_4 \cdot k_3 \epsilon_4 \cdot k_2 \epsilon_2 \cdot k_3 + k_2 \cdot k_3 \epsilon_2 \cdot k_4 \epsilon_4 \cdot k_3 - k_4 \cdot k_2 \epsilon_4 \cdot k_3 \epsilon_2 \cdot k_3]\} \\
& + \{2 \leftrightarrow 3\} + \{3 \leftrightarrow 4\} \qquad (7)
\end{aligned}$$

(ϵ_0 is the polarization of the Υ), and

$$\begin{aligned}
\epsilon_{4\mu} C^\mu_{J=0}(1) &= \sqrt{\tfrac{1}{6}}\, [\epsilon_1 \cdot \epsilon_4 k_1 \cdot k_4 - \epsilon_1 \cdot k_4 \epsilon_4 \cdot k_1] \left(m_\chi^2 + k \cdot k_4 - k_4^2\right), \\
\epsilon_{4\mu} C^\mu_{J=1}(1) &= \tfrac{1}{2} m_\chi k_4^2\, \varepsilon_{\alpha\beta\gamma\delta}\, e^\alpha \epsilon_4^\beta \epsilon_1^\gamma k_1^\delta, \qquad (8) \\
\epsilon_{4\mu} C^\mu_{J=2}(1) &= \sqrt{\tfrac{1}{2}} m_\chi^2 \left(k_1 \cdot k_4 \epsilon_1^\alpha \epsilon_4^\beta + k_4^\alpha k_1^\beta \epsilon_1 \cdot \epsilon_4 - k_1^\alpha \epsilon_4^\beta \epsilon_1 \cdot k_4 - k_4^\alpha \epsilon_1^\beta \epsilon_4 \cdot k_1\right) e^{\alpha\beta}.
\end{aligned}$$

e^α is a spin-1 polarization vector and $e^{\alpha\beta}$ is a spin-2 polarization tensor. For convenience the virtual gluon is labeled in Eqs. (6)–(8) by polarization ϵ_4 and momentum k_4 ($k_4 = P - k_2 - k_3 = k + k_1$). Terms which vanish due to the on-shell conditions $\epsilon_i \cdot k_i = 0$ ($i = 1, 2, 3$) and $\epsilon_0 \cdot P = 0$ have been dropped.

The overall factors in Eq. (3) are such that:

$$\begin{aligned}
& \mathcal{F}_{1J}(\mu) + (2J+1)\frac{16}{27\pi} \ln\left(\frac{\mu}{\mu_0}\right) \mathcal{F}_8 \equiv \\
& \quad 3 \int d[\Phi_4] \sum_{\text{spins}} \left[\mathcal{M}_J^2(2,3;1) + 2\mathcal{M}_J(2,3;1)\mathcal{M}_J(1,3;2)\right], \qquad (9)
\end{aligned}$$

where Φ_n denotes (infrared-cutoff) n-body phase space, normalized according to

$$\Phi_n[P \to p_1, \ldots, p_n] \equiv \int \prod_{i=1}^n \frac{d^3 p_i}{2E_i} \delta^4(P - \sum_i p_i). \qquad (10)$$

The factor of 3 on the right hand side of Eq. (9) accounts for symmetrization of $\mathcal{M}_J(2,3;1)$ under gluon label interchanges $1 \leftrightarrow 2$ and $1 \leftrightarrow 3$, taking account of the symmetry in the three gluon phase space.

The infrared divergence comes entirely from the first term in square brackets in Eq. (9), and is due to the P-wave charm quark propagator $1/(k \cdot k_1)^2$ in Eq. (6). It is therefore advantageous to organize the four-body phase space integral in Eq. (9) by taking the invariant mass of the χ_{cJ} and gluon "1" as one integration variable [8]

$$\int d[\Phi_4] = \int_0^{(m_\Upsilon - m_\chi)^2} d(k_{23}^2) \int_{m_\chi^2 + 2\mu_0 m_\chi}^{(m_\Upsilon - m_{23})^2} d(k_{1\chi}^2) \\ \times \Phi_2[P \to k_{23}, k_{1\chi}]\, \Phi_2[k_{23} \to k_2, k_3]\, \Phi_2[k_{1\chi} \to k_1, k], \tag{11}$$

where $m_{23}^2 \equiv k_{23}^2$. Note the infrared cutoff μ_0 on the energy of gluon "1" in the rest frame of the χ_{cJ}.

The infrared logarithm on the right-hand side of Eq. (9) can now be identified analytically by observing that $B_\mu(2,3) C_J^\mu(1)$ in Eq. (6) is given by a sum of terms each containing exactly one factor of k_1, if $k_4 = P - k_2 - k_3$ is used to eliminate the virtual gluon momentum. With this convention, one has

$$\sum_{\text{spins}} \mathcal{M}_J^2(2,3;1) = \frac{\gamma_J(k_1; P, k, k_2, k_3)}{(k \cdot k_1)^2}, \tag{12}$$

where k_1 appears explicitly in the function $\gamma_J(k_1; P, k, k_2, k_3)$ only in the combination $k_1/k \cdot k_1$. $\mathcal{F}_8$ is then given in terms of a manifestly infrared-finite three-body phase space integral, taking account of the fact that $\Phi_2(k_{1\chi} \to k_1, k) = \frac{1}{4} k \cdot k_1 / k_{1\chi}^2 \int d\Omega_{1\chi}^*$, where $\Omega_{1\chi}^*$ is the center-of-mass solid angle of the two body system:

$$(2J+1)\mathcal{F}_8 = \frac{27\pi}{32 m_\chi^2} \int_0^{(m_\Upsilon - m_\chi)^2} d(k_{23}^2)\, \Phi_2[P \to k_{23}, k] \\ \times \Phi_2[k_{23} \to k_2, k_3] \int d\Omega_{1\chi}^*\, \gamma_J(\widetilde{k}_1; P, k, k_2, k_3), \tag{13}$$

where

$$\widetilde{k}_1 \equiv \lim_{k \cdot k_1 \to 0} \frac{k_1}{k \cdot k_1}. \tag{14}$$

The finite four-vector $\widetilde{k}_1$ is readily expressed directly in terms of k_{23}^2 and $\Omega_{1\chi}^*$. An expression for $\mathcal{F}_{1J}$ can be obtained from Eqs. (9) and (13) by analogy with the identity $\int dx f(x)/x = f(0) \ln x + \int dx [f(x) - f(0)]/x$.

The contraction of currents and sum over polarizations in Eqs. (6) and (9) were performed symbolically using REDUCE [9] (leading to lengthy expressions, particularly for $J = 2$). The χ_{cJ} spin sums were done using (see e.g. Ref. [7]):

$$\sum_e e_\mu e_\nu = -g_{\mu\nu} + \frac{k_\mu k_\nu}{m_\chi^2} \equiv \mathcal{P}_{\mu\nu}, \\ \sum_e e_{\mu\nu} e_{\alpha\beta} = \tfrac{1}{2}\left[\mathcal{P}_{\mu\alpha}\mathcal{P}_{\nu\beta} + \mathcal{P}_{\mu\beta}\mathcal{P}_{\nu\alpha}\right] - \tfrac{1}{3}\mathcal{P}_{\mu\nu}\mathcal{P}_{\alpha\beta}. \tag{15}$$

Figure 2: Color-octet form factor $\mathcal{F}_8$ as a function of m_χ/m_Υ.

The phase space integrals were evaluated numerically using VEGAS [10]; modest integration grids are found to give very good convergence. The fact that $\mathcal{F}_8$ should be independent of J provides a stringent check of these calculations, given that the three currents C_J^μ have very different structures [cf. Eq. (8)]. This was verified explicitly in numerical calculations of Eq. (13), to better than a few tenths of a percent for all m_χ/m_Υ on a modest integration grid. Figure 2 shows the numerical results for $\mathcal{F}_8$ over a range of hypothetical meson masses. In Fig. 3 results for $\mathcal{F}_{1J}(\mu)$ are shown using a factorization scale $\mu = m_\Upsilon$.

The available experimental data on charmonium production in Υ decay is for the J/ψ:

$$B_{\rm exp}(\Upsilon \to J/\psi + X) \left\{ \begin{array}{ll} = (1.1 \pm 0.4) \times 10^{-3} & \text{CLEO [11]}, \\ < 1.7 \times 10^{-3} & \text{Crystal Ball [12]}, \\ < 0.68 \times 10^{-3} & \text{ARGUS [13]}. \end{array} \right. \tag{16}$$

An upper bound on H_8' can be extracted from this data by computing the "indirect" production of J/ψ due to the χ_{cJ} states. Assuming that radiative cascades from χ_{c1} and χ_{c2} dominate, with branching fractions $B_{\rm exp}(\chi_{c1} \to \gamma J/\psi) \approx 27\%$ and $B_{\rm exp}(\chi_{c2} \to \gamma J/\psi) \approx 13\%$ [6], the results presented here give:

$$H_8'(m_\Upsilon) \approx \left\{ \frac{\sum_J B(\Upsilon \to \chi_{cJ} + X' \to J/\psi + X)}{2.9 \times 10^{-5}} + 1.4 \right\} \text{MeV}. \tag{17}$$

The first number in brackets above comes from the color-octet subprocess rate $\hat{\Gamma}_8$, and the second number from the color-singlet rate $\hat{\Gamma}_1$. The experimental value for the total width $\Gamma_{\rm tot}(\Upsilon) \approx 52$ keV [6] was used, along with $\alpha_s(m_\Upsilon) \approx 0.179$ [1], and the values of H_1 and G_1^Υ given above.

Figure 3: Color-singlet form factors $\mathcal{F}_{1J}$ as functions of m_χ/m_Υ: $J = 0$ (short-dashed line), $J = 1$ (long-dashed line), $J = 2$ (solid line). The form-factors were evaluated using a factorization scale $\mu = m_\Upsilon$.

Equation (17) yields the bound $H_8'(m_\Upsilon) \lesssim 25$ MeV using the ARGUS upper limit, which is consistent with the other measurements. This bound is considerably larger than an estimate $H_8'(m_b) \approx 3$ MeV based on B meson decays [3],[‡] although a calculation of next-to-leading order QCD corrections to the color-singlet component of $B \to \chi_{cJ} + X$ is required before an accurate determination of H_8' can be made in that case [3].

This raises the possibility of significant direct production of J/ψ in the decay of the Υ, unless the branching fraction turns out to be considerably smaller than the ARGUS bound. Mechanisms for direct $\Upsilon \to J/\psi + X$ in perturbative QCD were first discussed in Refs. [14] and [15]. The direct production rate is suppressed by $O(\alpha_s^2)$ compared to the P-wave color-octet production mechanism considered here. However, the nonperturbative matrix elements which enter into P-wave production are of $O(v^2)$ relative to the corresponding parameter for S-wave production.

The full $O(\alpha_s^6)$ perturbative QCD amplitude for direct $\Upsilon \to J/\psi + X$ was recently evaluated in Ref. [16], corresponding to one-loop diagrams for $\Upsilon \to J/\psi + gg$, and tree diagrams for $\Upsilon \to J/\psi + gggg$. The $O(\alpha_s^2\alpha^2)$ electromagnetic amplitude for the two gluon decay mode was also evaluated. Unfortunately, only a crude estimate of the required phase space integrations was made in Ref. [16] (there is a costly convolution with a numerical calculation of the loop integrals for $\Upsilon \to J/\psi + gg$). Nevertheless, the calculation of Ref. [16] suggests a branching fraction for direct production of a few $\times 10^{-4}$. This would lead to a considerable reduction in the bound on H_8' extracted from Eqs. (16) and (17).

[‡] $H_8'(\mu)$ increases by only ≈ 0.3 MeV in the evolution from $\mu = m_b$ to $\mu = m_\Upsilon$ [cf. Eq. (2)].

To summarize, a complete calculation was made of the leading order rates for $\Upsilon \to \chi_{cJ} + X$, through both color-singlet P-wave and color-octet S-wave $c\bar{c}$ subprocesses. Experimental data on J/ψ production was used to obtain an upper bound on the nonperturbative parameter H_8', related to the probability for color-octet S-wave $c\bar{c}$ hadronization into P-wave charmonium. Improved experimental data, and a definitive calculation of the direct J/ψ production rate along the lines of Ref. [16], would allow for an accurate determination of H_8' from the results presented here.

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